IITians GATE CLASSES

Exclusive GATE Coaching by IIT/IISc Graduates

A division of PhIE Learning Center

QUICK REVISION

FORMULA SHEET

for

GATE -ME COMPRESSIBLE FLUID FLOW

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GATE Online Coaching

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OUR COURSES

Online Test Series

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More About IGC

SAMPLE STUDY MATERIALS
https://bit.ly/3Wu7lsp

SAMPLE MOCK TESTS
https://bit.ly/3FDyF0f

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https://bit.ly/3V4xU6j

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COMPRESSIBLE FLUID FLOW

REVIEW OF THERMODYNAMICS

1st Law:

$\delta q + \delta w = de$

$q \rightarrow$ Heat transfer, $w \rightarrow$ Work transfer

$e \rightarrow$ internal energy of the system per unit mass.

Adiabatic Process:

One in which no heat is added to or taken away from the system.

Reversible Process:

One in which no dissipative phenomenon occurs, that is, where the effect of viscosity thermal conductivity, and mass diffusion are absent.

Isentropic Process:

One that is both adiabatic and reversible.

$\delta q - pdv = de$ $\rightarrow$ For reversible process.

Where $p \rightarrow$ pressure,

$v \rightarrow$ Specific volume, $v = \frac{1}{\rho}$

$\rho \rightarrow$ density

Entropy and 2nd Law:

$$ds = \frac{\delta q_{rev}}{T}$$

where, $s \rightarrow$ Entropy per unit mass of the system.

$T \rightarrow$ Temperature of the system.

$$ds = \frac{\delta q}{T} + ds_{(irrev)} \rightarrow \text{more general form}$$

$ds_{irrev} \geq 0$

2nd Law: $ds \geq 0$

Tds Relations:

$Tds - pdv = de$

$Tds = de + pdv$

$\boxed{Tds = dh - vdp}$

$h \rightarrow$ specific enthalpy

$\boxed{h = c_p dT}$ $\boxed{e = c_v dT}$

here, $c_p \rightarrow$ Specific heat at constant pressure

$$c_p = \left(\frac{\partial h}{\partial T}\right)_{p=constant}$$

$c_v \rightarrow$ specific heat at constant volume

$$c_v = \left(\frac{\partial e}{\partial T}\right)_{v=constant}$$

Relationship Between c_P, c_V and R

$c_p - c_v = R$

$R \rightarrow$ Gas constant

$$R = \frac{R_u}{MW}$$

$R_u \rightarrow$ Universal gas constant

$MW \rightarrow$ Molecular weight

$$\left.\begin{aligned} s_2 - s_1 &= c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) \\ s_2 - s_1 &= c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right) \end{aligned}\right\} \left(\begin{array}{c}\text{Valid for} \\ \text{Calorically} \\ \text{perfect gas}\end{array}\right)$$

$$\gamma = \frac{c_p}{c_v} \rightarrow \text{Ratio of specific heats}$$

Isentropic Relations:

$s_1 - s_1 = 0$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Compressibility:

Isothermal compressibility:

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{T=constant}$$

Isentropic Compressibility:

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_{s=constant}$$

$$\tau_s = \frac{1}{\rho}\frac{d\rho}{dp}$$

$$d\rho = \rho\tau dp$$

Mach Number:

$$M = \frac{V}{a}$$

$V \rightarrow$ Local speed (Velocity) of the fluid Particle.

$a \rightarrow$ Local speed of sound.

Equation of State: $p = \rho RT$

Speed of Sounds:

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s=constant}}$$

$$a = \sqrt{\gamma RT} = \sqrt{\frac{\gamma p}{\rho}}$$

$\rightarrow$ valid for calorically perfect gas

Forms of energy equation for Steady Adiabatic Flows:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = \text{Constant}$$

For 1-D Adiabatic steady flows

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

$$h + \frac{u^2}{2} = h_o$$

$$\frac{T_o}{T} = \left(1 + \frac{\gamma - 1}{2}M^2\right)$$

Note: This is valid for steady adiabatic flow of a calorically perfect gas.

Isentropic Flow Relations for Steady Flow:

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}$$

Characteristic Mach Number:

$$M^* = \frac{u}{a^*}$$

$u \rightarrow$ Local speed of the fluid (1- D)

$a^* \rightarrow$ Characteristic speed of sound.

$$M^* = \frac{(\gamma+1)M^2}{2 + (\gamma-1)M^2};$$

$M \rightarrow$ Local Mach number

$M^* < 1$ if $M < 1$

$M^* = 1$ if $M = 1$

$M^* > 1$ if $M > 1$

$$M^* \rightarrow \sqrt{\frac{\gamma+1}{\gamma-1}} \text{ if } M \rightarrow \infty$$

Governing Equation for Standing Normal Shockwave:

Continuity : $\rho_1 u_1 = \rho_2 u_2$

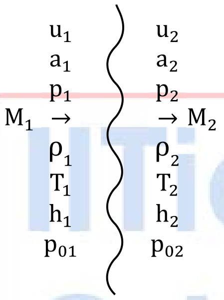
Momentum: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

Energy: $h_1 + \dfrac{u_1^2}{2} = h_2 + \dfrac{u_2^2}{2}$

$h = c_p T$

$P = \rho R T$

Property Change across the Standing Normal Shock Wave:



Shock Wave

Thickness of shockwave $\sim \dfrac{\nu}{a}$

$\nu \rightarrow$ Kinematic viscosity of the gas

$a \rightarrow$ Speed of sound

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right)$$

$$M_2^2 = \frac{1 + \left(\frac{\gamma-1}{2}\right)M_1^2}{\gamma M_1^2 - \left(\frac{\gamma-1}{2}\right)}$$

Limit Cases:

As $M_1 \rightarrow \infty$

$$\frac{\rho_2}{\rho_1} \rightarrow \frac{\gamma+1}{\gamma-1}$$

$$\frac{p_2}{p_1} \rightarrow \infty$$

$$\frac{T_2}{T_1} \rightarrow \infty$$

$$M_2 \rightarrow \sqrt{\frac{\gamma-1}{2\gamma}}$$

$T_{01} = T_{02}$

$$\frac{p_{02}}{p_{01}} = e^{-(s_2 - s_1)/R}$$

Measurement of Velocity in Compressible flow Using Pitot-Static Tube:

Subsonic Compressive Flow:

$$M_1^2 = \frac{2}{\gamma-1}\left[\left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$p_{01} \rightarrow$ Total pressure; $p_1 \rightarrow$ Static pressure

[Free stream conditions]

$$u_1^2 = \frac{2a_1^2}{\gamma-1}\left[\left(\frac{p_{01}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Quasi-One-Dimensional Flow:

Continuity:

$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$

$A \rightarrow$ Area at any section.

Momentum:

$$p_1 A_1 + \rho_1 u_1^2 A_1 + \int_{A_1}^{A_2} p\,dA = p_2 A_2 + \rho_2 u_2^2 A_2$$

Energy:

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

Differential Form:

$\rho u A = \text{Constant}$

$d(\rho u A) = 0$

Continuity:

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

Momentum:

$dp = -\rho u du$

Energy:

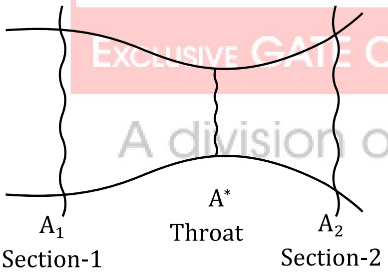
$dh + u du = 0$

Area Mach Number Relation:

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$$

Nozzle Flows: (C-D Nozzle)

$$\frac{A_1}{A_2} = \frac{M_2}{M_1}\sqrt{\left[\frac{1 + \frac{\gamma - 1}{2}M_1^2}{1 + \frac{\gamma - 1}{2}M_2^2}\right]^{\frac{\gamma+1}{\gamma-1}}}$$



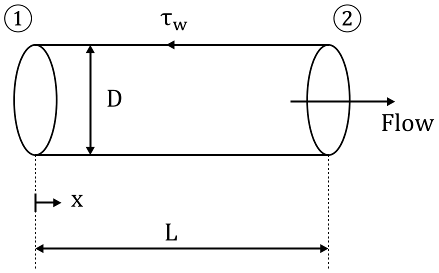
$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

Choked Mass Flow Rate at the Throat:

($u^* = a^*, M^* = 1$) At throat

$$\dot{m} = \frac{p_o}{\sqrt{RT_O}}\sqrt{\gamma}.\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} A^*$$

Flow with Friction:



$$\frac{T_2}{T_1} = \frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}$$

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\left[\frac{2 + (\gamma - 1)M_1^2}{2 + (\gamma - 1)M_2^2}\right]$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{T_1}{T_2}\right)$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{2 + (\gamma - 1)M_2^2}{2 + (\gamma - 1)M_1^2}\right]^{-1/2}$$

$$\frac{p_{02}}{p_{01}} = \frac{M_1}{M_2}\left[\frac{2 + (\gamma - 1)M_2^2}{2 + (\gamma - 1)M_1^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Choking in Frictional Flow:

$$\frac{T}{T^*} = \frac{\gamma + 1}{2 + (\gamma - 1)M^2}$$

$$\frac{p}{p^*} = \frac{1}{M}\left[\frac{\gamma + 1}{2 + (\gamma - 1)M^2}\right]^{1/2}$$

$$\frac{\rho}{\rho^*} = \frac{1}{M}\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{1/2}$$

$* \rightarrow$ Denotes properties at choking, i.e., where $M = 1$

$T \rightarrow$ Temperature upstream of the choking.

$M \rightarrow$ Mach number upstream of the choking

$p \rightarrow$ Pressure upstream of choking.

$\rho \rightarrow$ Density upstream of choking.

$$\frac{p_o}{p_o^*} - \frac{1}{M}\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]$$

$p_o \rightarrow$ Total pressure upstream of chocking.

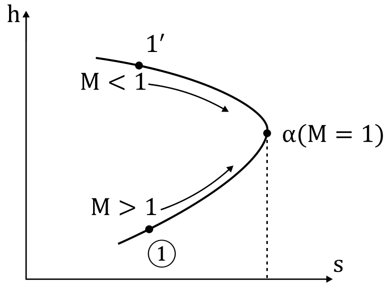
Average Friction Co-efficient:

$$\bar{f} = \frac{1}{L^*}\int_0^{L^*} f\,dx$$

L^* → Location from the origin where M = 1.

f → Local friction co-efficient f → f(x) depends on flow type.

Fano Curve:



Effect of Friction:

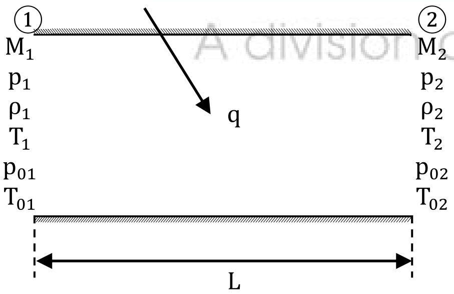
Variable	Subsonic region	Supersonic Region
Mach Number	Increases ($M_2 > M_1$)	Decreases ($M_2 < M_1$)
Pressure	decreases ($p_2 > p_1$)	Increase ($p_2 > p_1$)
Temperature	decreases ($T_2 < T_1$)	Increases ($T_2 > T_1$)
Total Pressure	decreases ($p_2 < p_1$)	decreases ($p_{02} < p_{01}$)
Velocity	Increases ($u_2 > u_1$)	decreases ($u_2 < u_1$)

Flow With Heat Addition:

Modified Energy Equation:

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

q → Heat transfer per unit mass.



$$\frac{p_2}{p_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$$

$$\frac{T_2}{T_1} = \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2 \left(\frac{M_2}{M_1}\right)^2$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{1+\gamma M_2^2}{1+\gamma M_1^2}\right)\left(\frac{M_1}{M_2}\right)^2$$

$$\frac{p_{02}}{p_{01}} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\gamma/\gamma-1}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2\left(\frac{M_2}{M_1}\right)^2\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right)$$

Choking Relations (at M = 1)

$$\frac{p}{p^*} = \frac{1+\gamma}{1+\gamma M^2}$$

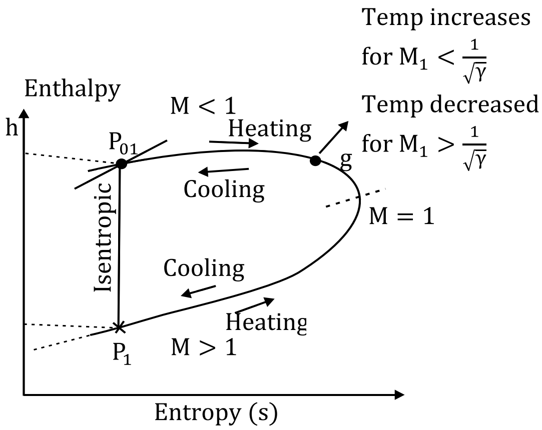
$$\frac{T}{T^*} = M^2\left(\frac{1+\gamma}{1+\gamma M^2}\right)^2$$

$$\frac{\rho}{\rho^*} = \frac{1}{M^2}\left(\frac{1+\gamma M^2}{1+\gamma}\right)$$

$$\frac{p_o}{p^*} = \frac{1+\gamma}{1+\gamma M^2}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\gamma/\gamma-1}$$

$$\frac{T_o}{T_o^*} = \frac{(\gamma+1)M^2}{(1+\gamma M^2)}\left[2+(\gamma-1)M^2\right]$$

Rayleigh Curve:



Effect of Flow with Heat Addition:

Variable	Subsonic Region	Supersonic Region
Mach Number	Increases ($M_2 > M_1$)	Decreases ($M_2 < M_1$)
Pressure	Decreases ($p_2 < p_1$)	Increases ($p_2 > p_1$)
Temperature	Increases for $M_1 < \gamma^{-1/2}$ And decrease for $M_1 > \gamma^{-1/2}$	Increases ($T_2 > T_1$)
Total Temperature	Increases ($T_{02} > T_{01}$)	Increases ($T_{02} > T_{01}$)
Total Pressure	Decreases ($p_{02} > p_{01}$)	Decreases ($p_{02} < p_{01}$)
Velocity	Increases ($u_2 > u_1$)	Decreases($u_2 < u_1$)

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